Let $N = \{0, 1, 2, \ldots \}$. Define $f: N \rightarrow N$ as computable.

- **Input:** $m 
- **Output:** $f(m)$

Let $C \subseteq N$ be computable (decidable).

- **Input:** $n 
  - **Yes:** if $n \in C$
  - **No:** if $n \notin C$

$(C$ codes decidable "problems")

A computably enumerable set $W \subseteq N$.

There is a computable function $f$ which enumerates it.

$W = \{ f(0) = w_0, f(1) = w_1, f(2) = w_2, \ldots \} = \{ w_0, w_1, w_2, \ldots \}$

- **Input:** $n 
- **Output:** yes if $n \in W$
- **Output:** no if $n \notin W$
\[ M \in \mathbb{W} \implies M \in \mathbb{K} \mathbb{W} \]

Unsolvability of the Halting Problem

\[ \exists e : e \in \mathbb{W} \mathbb{E}_3 = HS \]
\[ HS \notin X \]

reduce algorithmically

\[ \mathbb{W}_e \]

\[ \mathbb{W}_1, \mathbb{W}_2, \ldots, \mathbb{W}_n = \operatorname{dom} P_e = \{ x | P_e(x) = 1 \} \]

\[ P_0, P_1, P_2, \ldots \]

most sets are non-algorithmic

most sets are non-computable

\[ \implies \text{Halting Set} \]

\[ K = \{ e : e \notin \mathbb{W}_e \} \text{ not computable} \]

\[ P_e(e) \downarrow \]

\[ (\text{halt}) \]

Part: Assume otherwise

then \( \{ e : e \notin \mathbb{W}_e \} \text{ is computable} \)

so computably enumerable \( \exists e : e \notin \mathbb{W}_e = \mathbb{W}_m \)

1956-57: The priority method

(Friedberg and Muchnik)

compatible structure with certain non-computable property.

There is a compatible linear ordering of order type \( w \times w \) such that its \( w \)-part

is not computable (not even computably enumerable).

\[ \text{domain} = \mathbb{N} \]

\[ < \text{ is computable} \]

1931: Van der Wolda

Algebra field theoretic

algorithms are finite number

of steps
$R \neq W_0, W_1, W_2, \ldots$

Goal:

$W_0 = \emptyset \subseteq W_1 \subseteq W_2 \subseteq \ldots$

$W_{e,0} = \emptyset \subseteq W_e \subseteq W_{e,1} \subseteq \ldots$

Build a compatible ordering in stages

using $0, 1, 2, \ldots$

$\ldots \quad \text{w} \quad \ldots \quad \text{w}$

$\text{At each stage } \leq \text{ finite linear ordering}$

Stage 0: $\text{w} \quad \text{w} \quad \text{w}$

Cannot change the ordering of elements to assure that the ordering is compatible.
Priority Listing

No > Pe > N₁ > P₁ > N₂ > P₂ >

R

No, N₁

1

1

L

L

Note: Pe is attacked at this stage
No requirement of higher priority involved.

At the first two unused numbers from 0, 1, 2...
into the leftmost empty spot in R and
rightmost empty spot in R.

Assume m is enumerated as We.

If Ce has not been clearly defined,
Pc requires attention.

If m is not placed at 0, 1, 2, ... 5
6th place in R define Ce = m.

and if m is in R, then move m

AND all.

NUMBERS in R to the right of m
into the rightmost
places in R.

Finite Injury priority method

R ≠ We de 6 We = R

Gerald makes infinite injury theory
for infinite enumerable sets.